

## **SIMULATION OF COUPLED HYDROMECHANICAL PROBLEMS IN FRACTURED POROUS MEDIA**

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**Abstract.** *The high operational costs of the oil industry increased the need of numerical simulation. Modern simulators allow phenomena such as thermal effects, mechanics and fluid flow to be treated contributing to reduce exploitation risks. The improvement of computational resources as hardware and software has been responsible for complex and large scale simulations of three-dimensional sedimentary basin models in regional scale, allowing the analysis over the history of a sedimentary basin formation, from its sediments deposition up to generation, expulsion, migration, trapping and conservation of hydrocarbons. Geological faults can play an important role in producing migration paths and can be simulated by interface finite elements. In this work, the effects of mechanical simulation of stress of geological fractured media by zero-thickness interface elements (considering small displacements) are coupled to fluid flow in faults modeled by three-dimensional hydraulic interface elements based on the two-dimensional element developed by Segura et al. Besides the coupling through the storage equation, the fault aperture from mechanical analysis feeds the hydraulic system establishing potentially paths for oil migration.*

**Keywords:** *Flow in porous and fractured media, Hydromechanical coupled analysis*

## 1. INTRODUCTION

In a geomechanical sense, a discrete fracture represents a discontinuity that can occur in either homogeneous or heterogeneous geologic media (Selvadurai and Yu, 2005) with extensions that can reach the order of microfissures or thousands of kilometers (<http://fracflow.dk>). The presence of fracture in the upper brittle crust is due to instabilities from the lower ductile layers. These instabilities, characterized as either uplifts or subsidence phenomena, arise from a variety of geologic phenomena such as relative lithostatic movements, thermal gradients and fluid pressure (Tuncay and Ortoleva, 2002).

The effect of the fractures over the hydromechanical behavior plays an important role on understanding such processes like oil recovering, hydrogeology, pollutant dispersion and geotechnical applications in general. The concern on understanding of such coupled phenomena has been growing significantly in the last decade (Wan, 2002).

The relative displacements between fracture walls usually generates an aperture which can be understood as either a preferential path for fluid flow or not, according to the existence of a void or not. Thus, the aperture can be filled with material (coming out from wall friction or transported by fluid flow) and consequently presenting higher or lower permeability relative to the surrounding porous media, characterizing the formation of a joint (<http://fracflow.dk>; Selvadurai and Yu, 2005). The fracture existence as a void or filled with some material, which can have significant difference in either hydraulic or mechanical material properties from surrounding media, has a great impact over the original hydromechanical behavior of the formation.

In this work we study the discrete modeling of well characterized fractures in geomechanical scenarios by the finite element method. This study comprises the coupled hydromechanical behavior simulation of fractured porous media (Wan, 2002). The mechanical modeling is carried out by trilinear tetrahedral elements for the domain, whereas faults are represented by discrete joints as three-dimensional six-noded prismatic interface elements (Coutinho et al., 2003). The fault material presents elastoplastic behavior according to the Mohr-Coulomb theory (Crisfield, 1996). We consider small displacements only. The hydraulic phenomenon studied comprises one-phase fluid flow according to Darcy's law. In the hydraulic modeling, the domain is discretized by trilinear tetrahedral elements and three-dimensional prismatic six-noded interface elements, expanded from two-dimensional ones as proposed by Segura and Carol (2004), for saturated porous media, thus keeping the same mesh topology for both mechanical and hydraulic analyses. The main target is to build a coupling procedure between hydraulic and mechanical analyses through fractures, hence porosity changes and gravitational effects are neglected.

The hydromechanical coupling is accomplished by the storage equation (Verruijt, 2001) and fault apertures, considering steady state fluid behavior. The fault permeability is assumed constant and flow within the fault is laminar. As a great advantage, the same interface element topology for both mechanical and hydraulic modeling allows direct field interchange with no need of extra manipulations.

The code implemented was based on the high performance computing paradigm of shared memory processing and uses the data locality optimization concept through special data-reordering algorithms for finite element data (Ribeiro and Coutinho, 2005). We employ here the edge-based inexact-Newton method introduced by Coutinho et al. (2001) for solving the resulting set of coupled non-linear equations.

This work is organized as follows: section 1 presents the equations underlying the phenomena; in section 2, the governing equations are presented; section 3 establishes the methodology used to implement the coupling procedure; in section 4, we treat three examples

comprising different scenarios for different hydraulic boundary conditions and finally, section 5 presents the main conclusions and future work.

## 2. GOVERNING EQUATIONS

The proposed approach is based on the following governing equations of a quasi-static deformation and fluid flow in saturated porous matrix for a domain  $\Omega$  as,

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = 0 \quad (1)$$

$$\frac{\partial q_j}{\partial x_j} = 0 \quad (2)$$

Equations (1) and (2) are the momentum and mass balance for solid and fluid phases respectively;  $\sigma_{ij}$  is the Cauchy stress tensor,  $x_j$  is the position vector,  $\rho$  is the weight per unit volume,  $b_i$  represents body forces and  $q_j$  is the fluid flux. Equation (1) is subjected to the kinematic, traction and flux boundary conditions:

$$u_i(x,t) = \bar{u}_i(x,t) \text{ in } \Gamma_u; \sigma_{ij}n_j = h_i(x,t) \text{ in } \Gamma_h \quad (3)$$

$$p(x,t) = \bar{p}(x,t) \text{ in } \Gamma_p; q_jn_j = Q_j(x,t) \text{ in } \Gamma_Q \quad (4)$$

where  $\Gamma_u$  represents the portion of the boundary where displacements are prescribed ( $\bar{u}_i$ ) and  $\Gamma_h$  represents the portion of the boundary where tractions are specified ( $h_i$ ); for hydraulic counterpart,  $\Gamma_p$  and  $\Gamma_Q$  are the portions of the boundary where hydraulic pressures ( $\bar{p}$ ) and fluxes ( $Q_j$ ) are applied respectively;  $t$  represents an increment parameter only. The boundary  $\Gamma$  of the body is given by  $\Gamma_s = \Gamma_u \cup \Gamma_h$  and  $\Gamma_f = \Gamma_p \cup \Gamma_Q$  for solid and fluid boundary conditions respectively, with  $\Gamma \equiv \Gamma_s \equiv \Gamma_f$ .

Equations (2) and (4) are explicitly coupled through total stress relation given by (Verruijt, 2001)

$$\sigma_{ij} = \sigma_{ij}^e + \delta_{ij} p_p \quad (5)$$

where  $\sigma_{ij}^e$  is the effective stress field over the entire solid porous matrix and  $\delta_{ij}$  is Kronecker delta. The hydraulic pressure fluctuation due to sources, sinks and relative to initial pressure state, induces the correspondent volumetric deformation of solid phase, acting uniformly over the porous matrix and is evaluated as (Verruijt, 2001)

$$d\varepsilon_{kkp} = \delta_{ij} \frac{dp_p}{k_s} \quad (6)$$

where  $1/k_s$  is the rock compressibility, assumed constant for small displacements and is defined as (Lambe, 1979)

$$\frac{1}{k_s} = \frac{(1+\nu)(1-\alpha)}{E} \quad (7)$$

where  $E$  is the Young Modulus and  $\alpha$  is the Biot constant.

The constitutive equation that links effective stresses to solid phase deformations is independent from pore-pressure field. Despite other effects, one can write the incremental relation between stress and strain as (Crisfield, 1996)

$$d\sigma_{ij}^e = C_{ijkl}(d\varepsilon_{ij} - d\varepsilon_{kk_p}) \quad (8)$$

where  $d\varepsilon_{ij}$  is the total deformation of solid phase.

### 3. FINITE ELEMENT APPROACH

The finite element discretization of Eq. (1) and (4) generates the discrete equilibrium equation given by

$$\mathbf{F}_i - \mathbf{F}_e = \mathbf{0} \quad (9)$$

where  $\mathbf{F}_i$  and  $\mathbf{F}_e$  are the internal and external forces vector respectively. By considering the discretization of Eq. (1) and (3) along with incremental loading and the non-linear behavior of faults configurations of closing, sliding and opening, the linearization of these equations produces the set of non-linear equations to be solved incrementally by a Newton-like algorithm as

$$\mathbf{K}_T \Delta \mathbf{u} = \mathbf{F}_i - \mathbf{F}_e = \mathbf{R} \quad (10)$$

where the tangent stiffness matrix  $\mathbf{K}_T$  contains the contributions of both the solid elements and the kinematically consistent interface elements (Coutinho et al., 2003);  $\Delta \mathbf{u}$  is the displacement field and  $\mathbf{R}$  is the residual vector. Internal forces are evaluated considering the elastoplastic behavior of the faults and the elastic domain behavior, and external forces are evaluated as follows

$$\mathbf{F}_e = \int_{\Omega} \mathbf{B}^T N \alpha d p d\Omega - \int_{\Omega} \mathbf{B}^T C N d p / k_s d\Omega + \int_{\Omega} \mathbf{B}^T \sigma_g d\Omega - \mathbf{K} \bar{\mathbf{u}} \quad (11)$$

where  $N$  is the shape function vector and  $\mathbf{B}^T$  is the discrete divergent operator; the domain is split into two subdomains composed by the porous matrix and faults, that is  $\Omega = \Omega_p \cup \Omega_f$ . The first and second term of right hand side of Eq. (11) evaluate the dragging force and the hydrostatic deformation respectively, over the solid phase (porous matrix and fractures) due to pore-pressure field oscillations.

Diffusion effects through faults are dependent of aperture configurations and are evaluated according to Eq. (2) and (4) with constant permeability and apertures input from mechanical analysis. The coupling procedure implemented is depicted in Fig. 1.

The resulting set of coupled non-linear equations is solved by an edge-based inexact-Newton algorithm combined with the Preconditioned Conjugate Gradient (PCG) algorithm for solving the series of linearized system of equations. The preconditioner is diagonal and block-diagonal for the hydraulic and mechanical solvers respectively. For the inexact-Newton algorithm, the tolerances vary in a closed interval ranging between  $10^{-6}$  and  $10^{-3}$  for the mechanical solution. For the hydraulic solution, the tolerance is fixed to  $10^{-6}$ .

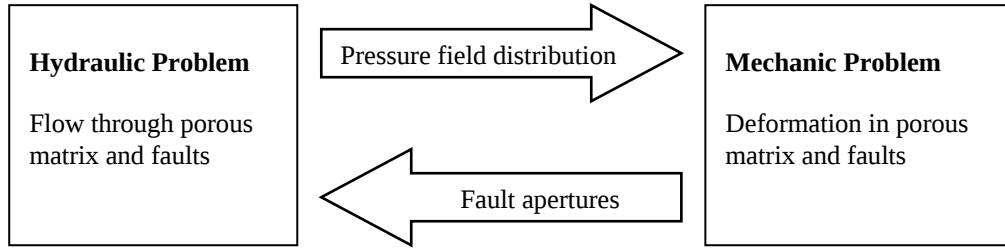


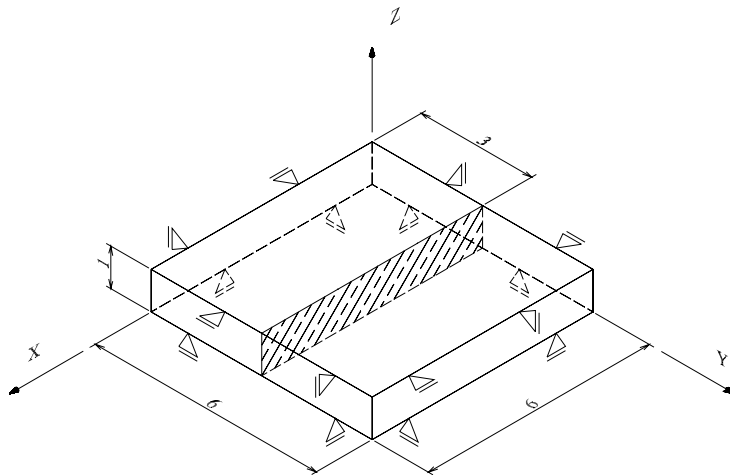
Figure 1 – Coupling procedure for hydromechanical analysis in fractured porous media for steady-state behavior.

#### 4. EXAMPLES AND RESULTS

The example presented is composed by a domain cut out by a fault, with dimensions of 6 by 6 length units and unitary thickness. The mechanical boundary conditions comprise normal displacements prescribed null in all planes unless the upper plane (where all displacements are free) and hydraulic pressure applied incrementally. The objective is to study the interaction between the fault configuration and the pressure and displacement field history according to the loading increment regime. In order to simplify, all results comprised the solid phase configuration after equilibrium of Eq. (10) for the first pressure field only.

Figure 2 presents the domain dimensions, mechanical boundary conditions and material, where  $k_x$ ,  $k_y$  and  $k_z$  is the permeability for each direction; at fault,  $\phi$  is the internal friction angle,  $c$  is the cohesion and  $K_s$ ,  $K_t$  and  $K_n$  is the stiffness coefficient related to local directions –  $s$  and  $t$  as tangential and  $n$  as normal to interface element mid plane. The Biot coefficient is assumed to be the same for the domain and the fault.

The domain was discretized into 225,114 four-noded tetrahedral and 1,200 six-noded interface elements, generating 42,458 nodes. All runs were carried out in a Pentium IV, 2.66 GHz and RAM 512 MB.



Material Properties for Domain:

$$\begin{aligned} E &= 5 \times 10^8 [F/L^2] \\ \nu &= 0.29 \\ \alpha &= 0.85 \\ k_x = k_y = k_z &= 10^{-5} [L/T] \end{aligned}$$

Material Properties for Fault:

$$\begin{aligned} K_s = K_t = K_n &= 10^8 [F/L^3] \\ \phi &= 30^\circ \\ c &= 10^6 \\ k_x = k_y &= 10^{-3} [L/T] \\ k_z &= 10^{-3} [1/T] \end{aligned}$$

Figure 2 – Domain dimensions, mechanic boundary conditions and materials.

The analysis was divided into 3 cases according to hydraulic boundary condition for pressure. The objective is to observe the response of fault configuration under pressure action and deformations imposed by dragging and hydrostatic deformations. The coordinate points at (6,3,1), (3,3,1) and (0,3,1) – named points 1, 2 and 3 respectively – have pressure and aperture values plotted against pressure step number to locally evaluate the field trend behavior along load steps.

#### 4.1 Case 1

For this case, the boundary planes located at  $x = 0$ ,  $y = 0$  and  $y = 6$  are prescribed with null pressure generating 120,886 and 40,434 mechanic and hydraulic equations respectively. The fault line located at coordinate  $xyz = (6,3,z)$  is a source with prescribed pressure ranging on 2 increments of  $5 \times 10^{-4}$ ,  $2.5 \times 10^{-3}$ , and 18 increments starting with  $5 \times 10^{-3}$  up to  $9 \times 10^{-2}$  up to through uniform steps of  $5 \times 10^{-3}$ . Figure 3 presents pressure field distribution over the deformed domain for steps 1 to 12. The displacement field is amplified 40 times to improve its visualization.

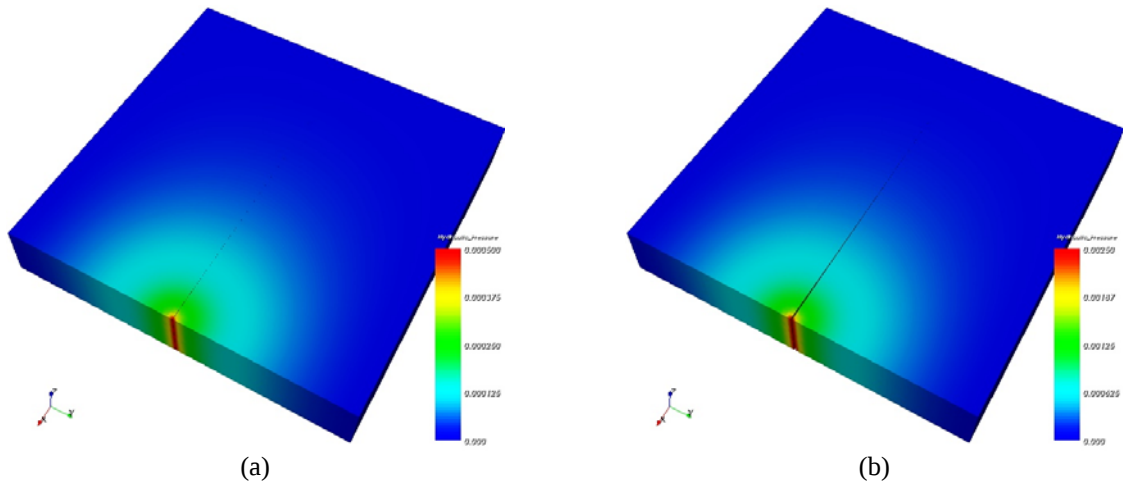


Figure 3.1 – Pressure field distribution over deformed domain for case 1, steps 1 (a) and 2 (b).

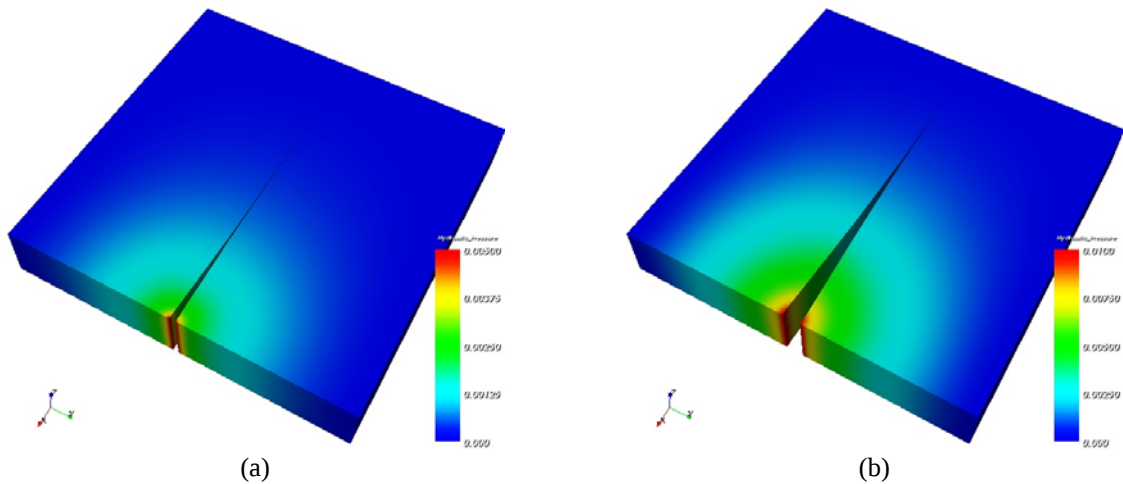


Figure 3.2 – Pressure field distribution over deformed domain for case 1, steps 3 (a) and 4 (b).

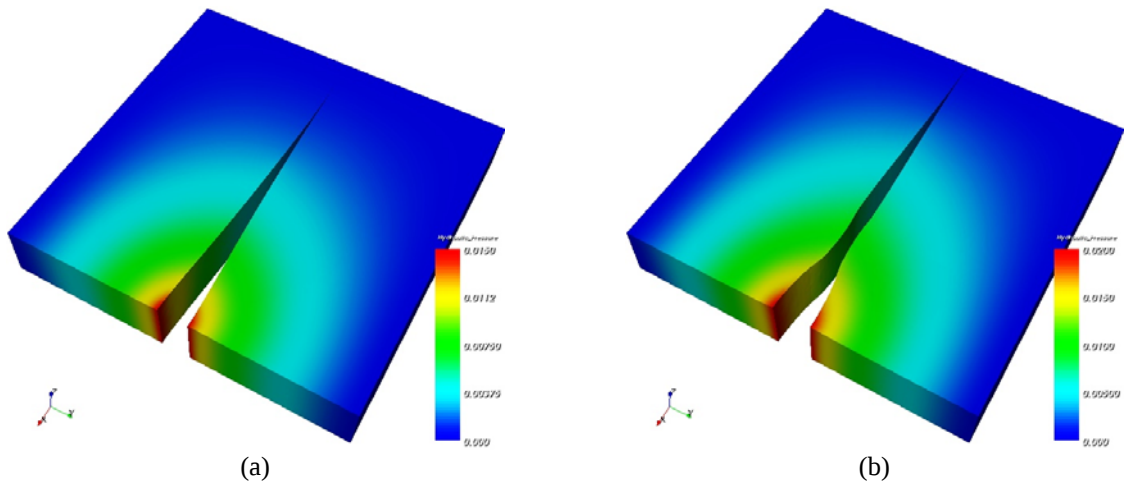


Figure 3.3 – Pressure field distribution over deformed domain for case 1, steps 5 (a) and 6 (b).

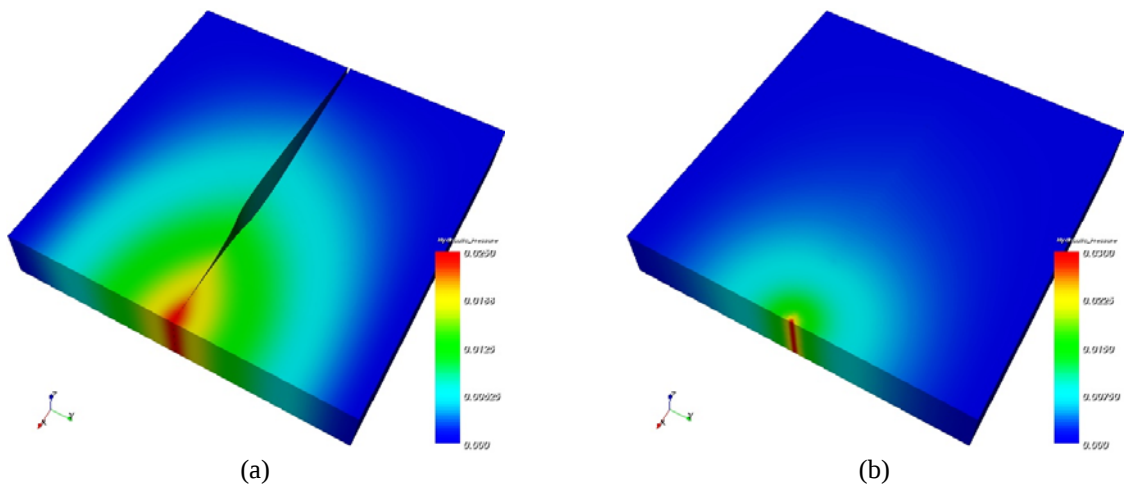


Figure 3.4 – Pressure field distribution over deformed domain for case 1, steps 7 (a) and 8 (b).

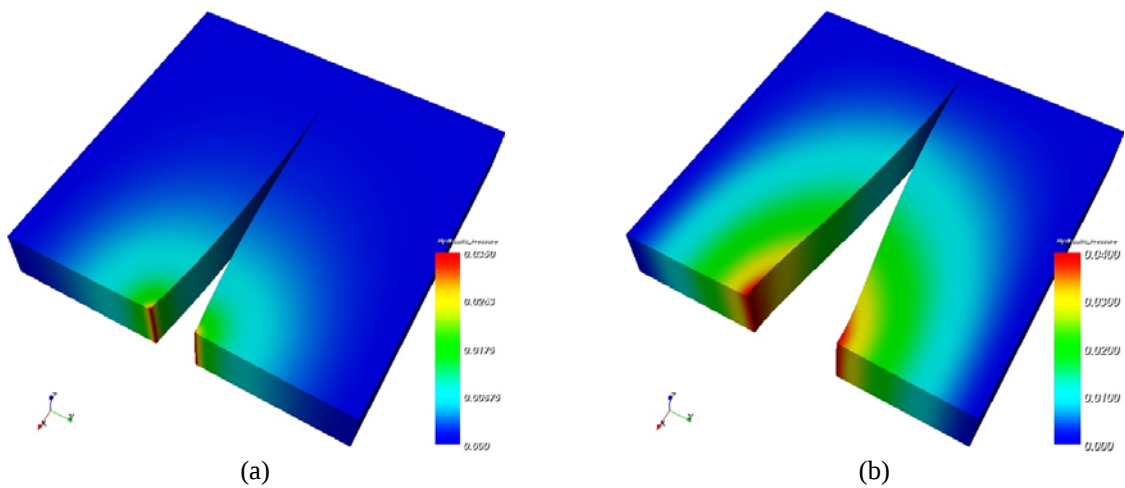


Figure 3.5 – Pressure field distribution over deformed domain for case 1, steps 9 (a) and 10 (b).

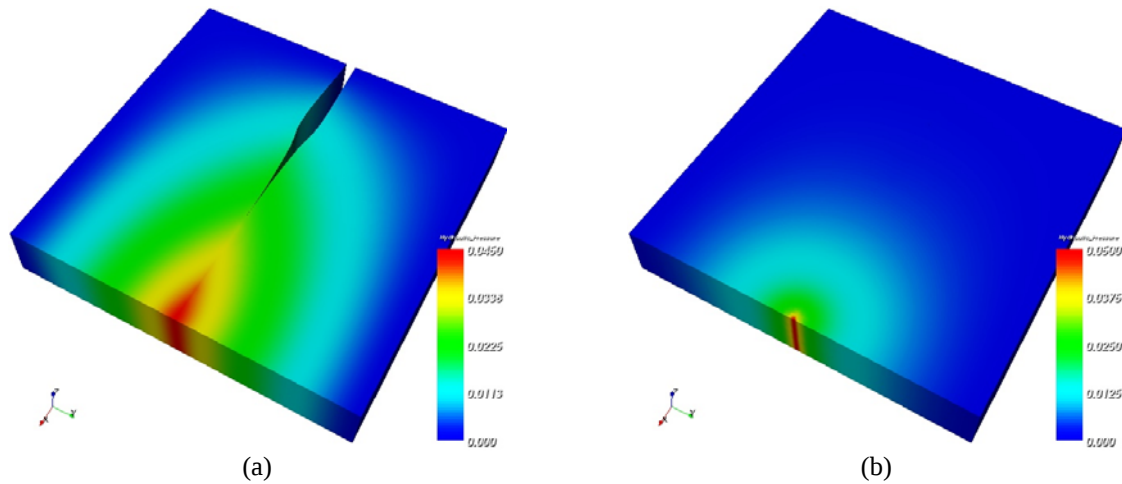


Figure 3.6 – Pressure field distribution over deformed domain for case 1, steps 11 (a) and 12 (b).

According to Fig. 3, pressure field is affected as aperture fault increases. Step 11 clearly pictures the stretching of pressure field along the fault. The fault behavior is characterized by an open-close cycle. As can be seen in step 6, an initial deformation tends to diminish the aperture, going through until it is completely closed in step 8. The cycle restart in step 9 ending with the fault completely closed in step 12. This cycle has similar behavior until step 20, performing the same period of 4 load increments to complete one cycle.

This trend to close the fault is originated by a combination of dragging and hydrostatic deformation over the domain, which is significant, since the boundary condition drives this pressure spreading along the plane at  $x = 0$ . Searching for the first step that returns to the initial fault configuration, the jump from step 7 to 8 reveals that whenever the fault aperture is greater than zero at boundary, there is a sudden unbalanced fluid flux that has a tendency to abruptly close the fault. This behavior can be seen again in the jump from step 11 to 12. The same phenomenon occurred in jumps on steps 14 to 15, 17 to 18, and it is forecasted to happen during the jump from step 20 to 21.

Figure 4 depicts the pressure and aperture at points 1, 2 and 3 for each pressure load step. Comparing figures (a) and (b), one can see that, for point 1, the linear pressure increasing goes along with the open-close cycle. This behavior is more evident at point 2, in which the pressure and aperture present similar behavior, with pressure increasing together with fault opening and with pressure decreasing together with the fault closing. As pressure load steps go through, it is clear that the aperture increases with pressure increasing.

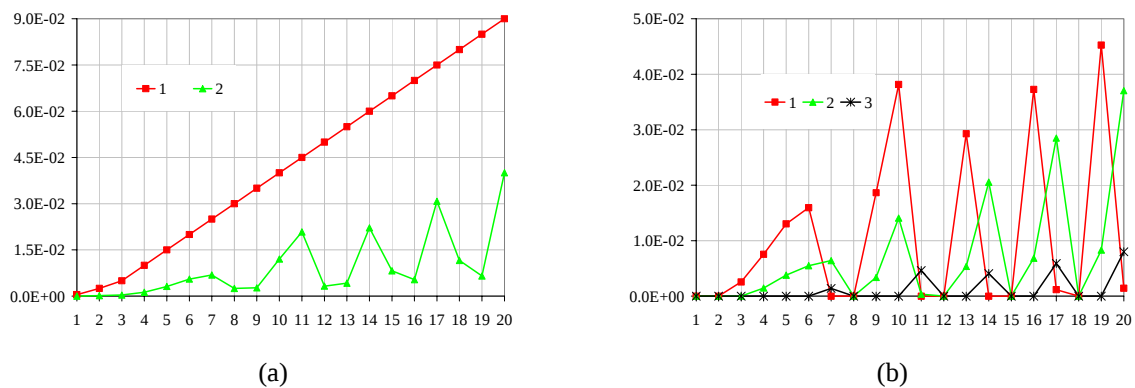


Figure 4 – Pressure versus step number (a); fault aperture versus step number (b) in points 1, 2 and 3 for case 1.

The complete mechanical analysis was performed in 226 non-linear iterations, generating an average of 203 PCG iterations per non-linear iteration; hydraulic analysis needed a total of 6,989 PCG iterations. Figure 5 depicts mechanical PCG iterations against load step number and interface elements opened against step number accumulated from total non-linear iterations number of each load step. It is clear the increasing/decreasing on the number of PCG iterations for both analyses, as the number of interface elements opened increases/decreases, as the fault aperture deteriorates the system condition number.

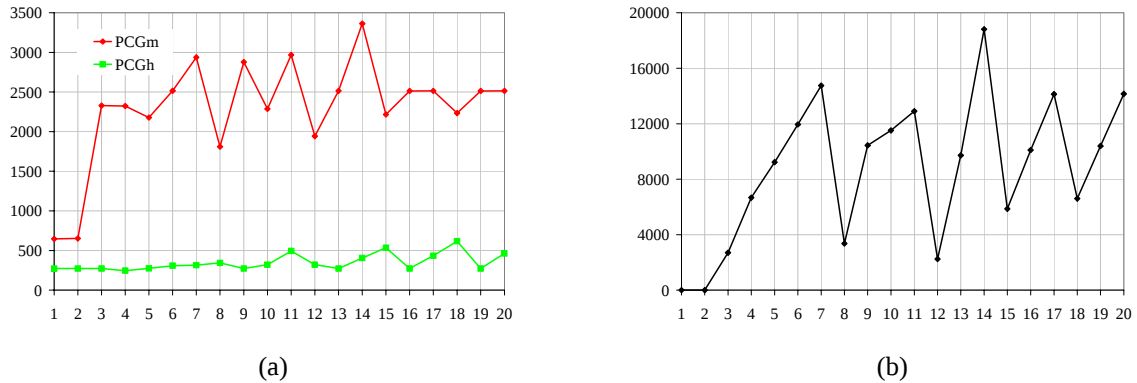


Figure 5 – PCG iterations accumulated number versus step number (a); interface elements opened accumulated number versus step number (b).

## 4.2 Case 2

In this case, there is a source and sink lines located at coordinates  $xyz = (6,3,z)$  and  $(0,3,z)$  respectively, with pressure equally prescribed but with opposite sign, ranging from  $5 \times 10^{-3}$  up to  $10^{-1}$  in 20 uniform steps of  $5 \times 10^{-3}$ . There were generated 120,886 and 42,414 mechanic and hydraulic equations respectively. Figure 6 presents pressure field distributions over the deformed domain for steps 1 to 8. The displacement field is amplified 40 times for a better visualization.

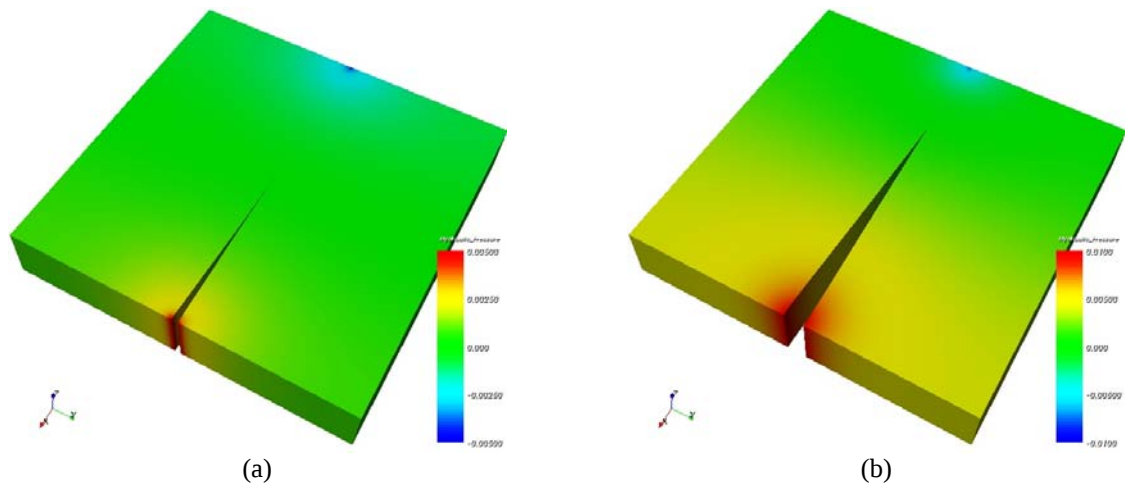


Figure 6.1 – Pressure field distribution over deformed domain for case 2, steps 1 (a) and 2 (b).

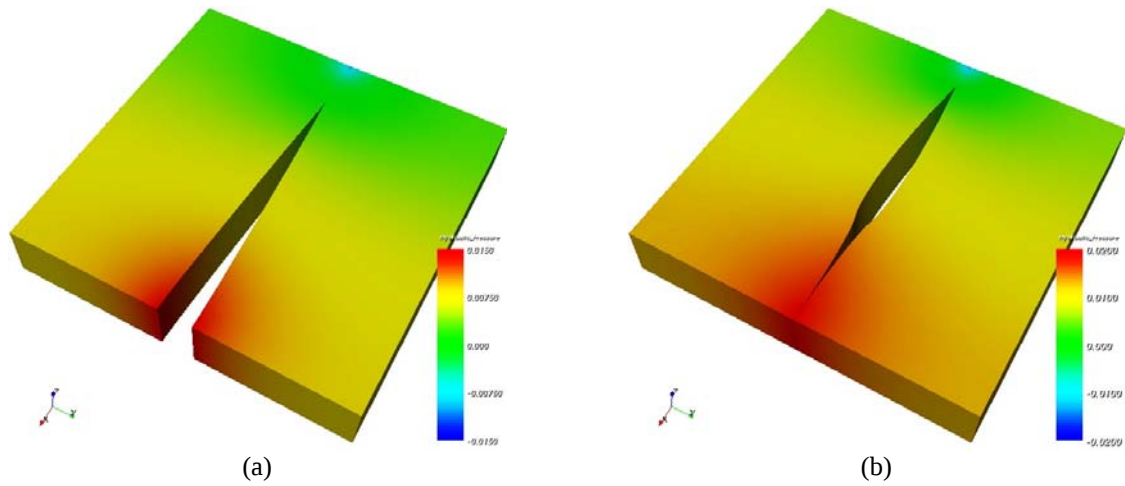


Figure 6.2 – Pressure field distribution over deformed domain for case 2, steps 3 (a) and 4 (b).

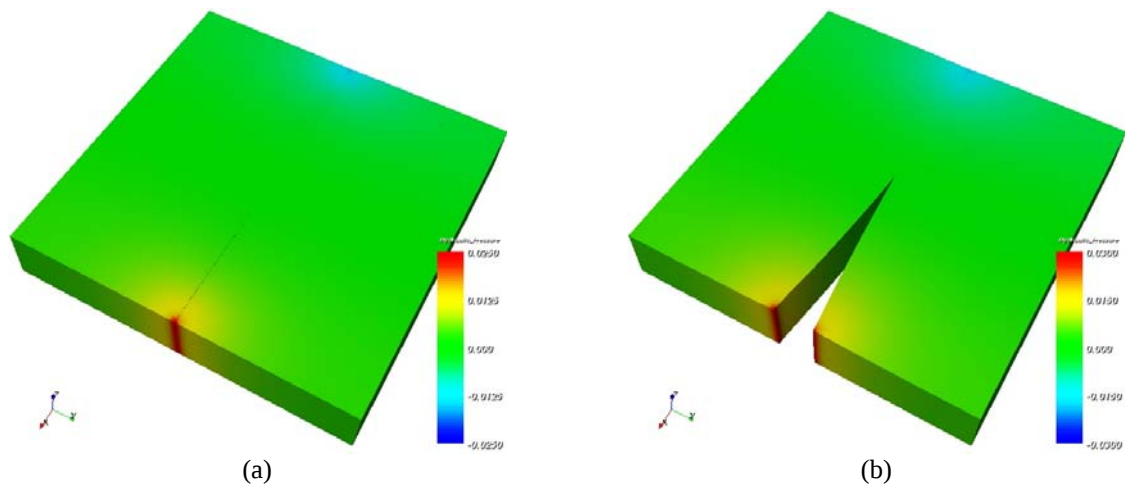


Figure 6.3 – Pressure field distribution over deformed domain for case 2, steps 5 (a) and 6 (b).

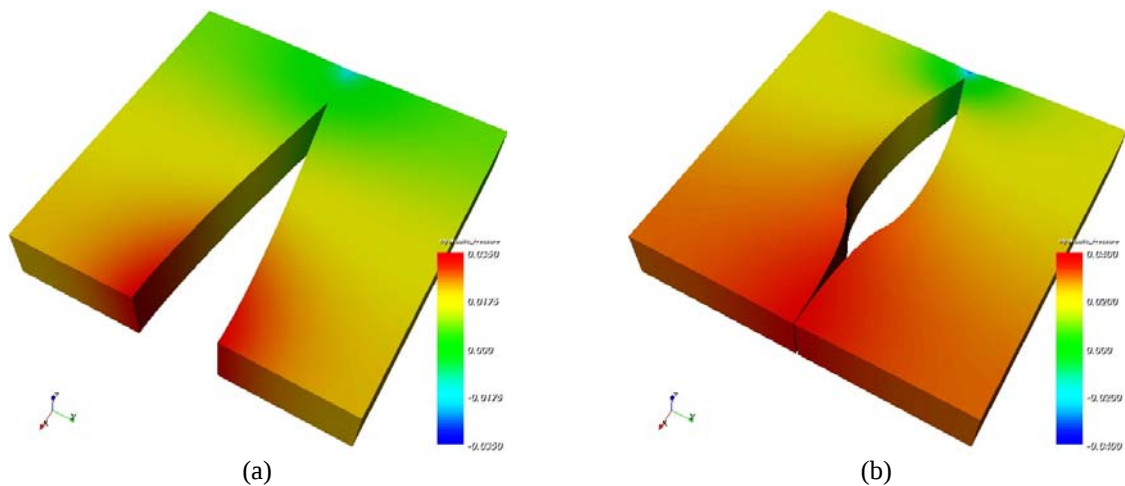


Figure 6.4 – Pressure field distribution over deformed domain for case 2, steps 7 (a) and 8 (b).

In this example, the subpressure at sink produces an advance of complete fault closing, as can be seen in the jump from step 4 to 5. This behavior also happens in the jump from step 8 to 9. From step 10 and ahead, point 3 presents aperture for steps 12, 16 and 20 with stride 4, as presents Fig. 7(b). In steps 12 and 16, the significant aperture generates sharp pressure jumps for the next steps, as can be seen in Fig. 7(a).

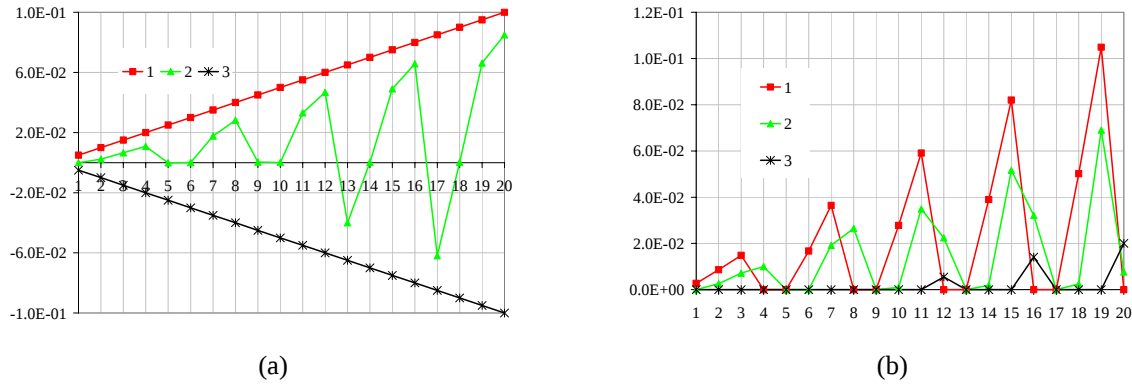


Figure 7 – Pressure versus step number (a); fault aperture versus step number (b) in points 1, 2 and 3 for case 2.

The complete mechanical analysis was performed in 236 non-linear iterations, generating an average of 206 PCG iterations per non-linear iteration; hydraulic analysis needed a total of 14, 935 PCG iterations. It can be seen in Fig. 8 that the maximum accumulated number of active fault elements is at step 8, generating the greatest number of mechanical PCG iterations (PCGm). Afterwards, the fault configuration presents a stable cycle until step 20, yielding a stable cycle of PCGm iterations. However, the number of PCG hydraulic (PCGh) iterations increases in periods of 4 steps. This behavior is due to the interface aperture increasing and then contributing to deteriorate the condition number of the hydraulic linear systems. That is, the mechanical system is dependent on the interface elements opened only, whereas the hydraulic system, besides this number, is also dependent of the aperture values.

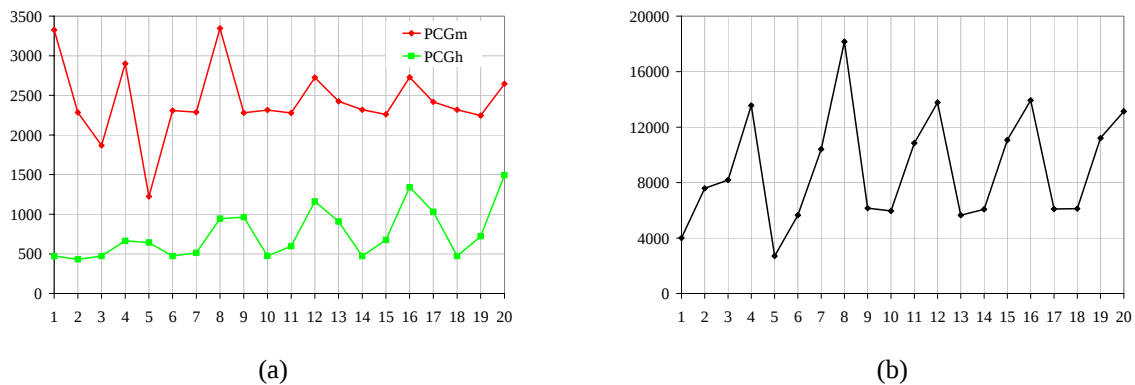


Figure 8 – PCG iterations accumulated number versus step number (a); interface elements opened accumulated number versus step number (b).

### 4.3 Case 3

This case deals with a source line located at coordinate  $xyz = (6,3,z)$  with 20 pressure load increments uniformly applied from  $10^{-3}$  up to 0.02. There is a sink line with null pressure applied at coordinates  $xyz = (0,3,z)$ . There were generated 120,886 and 42,414 mechanical and hydraulic equations respectively. Figure 9 presents the pressure field distribution over the deformed domain for steps 1 to 8. The displacement field is amplified 40 times for a better comprehension.

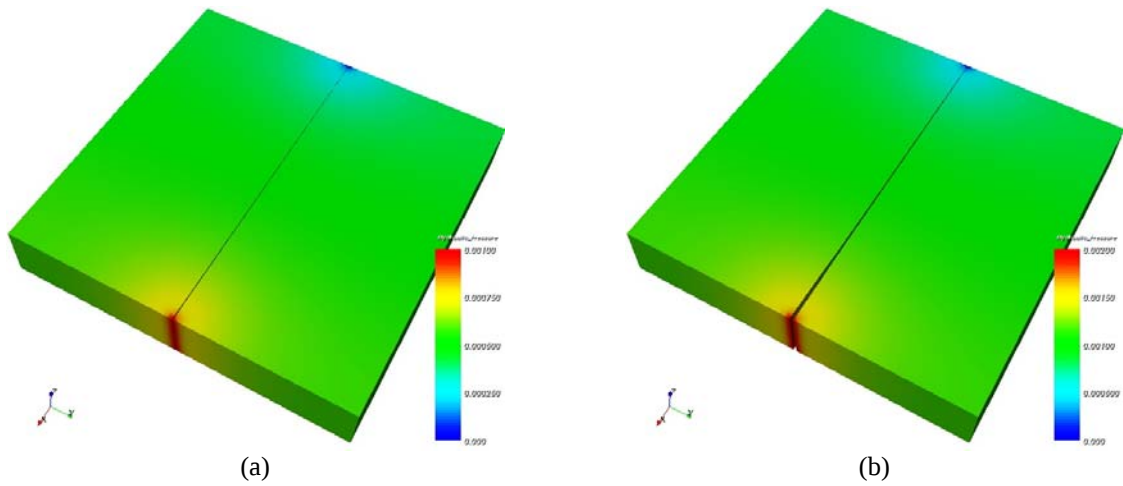


Figure 9.1 – Pressure field distribution over deformed domain for case 3, steps 1 (a) and 2 (b).

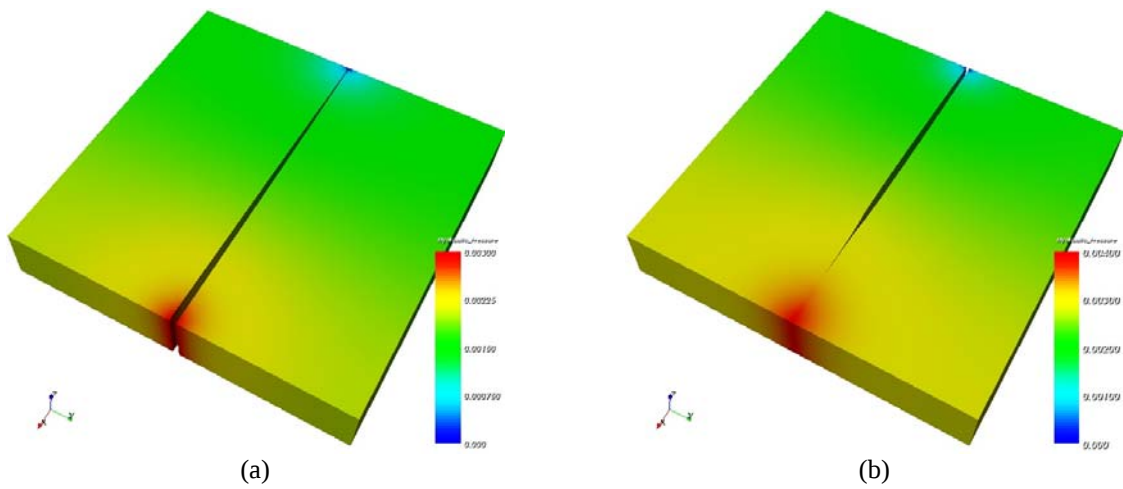


Figure 9.2 – Pressure field distribution over deformed domain for case 3, steps 3 (a) and 4 (b).

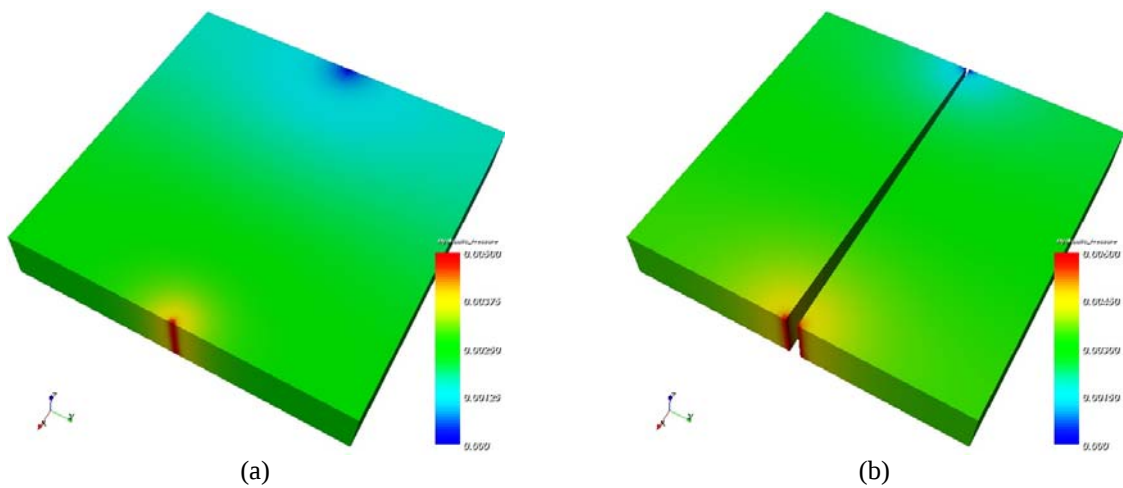


Figure 9.3 – Pressure field distribution over deformed domain for case 3, steps 5 (a) and 6 (b).

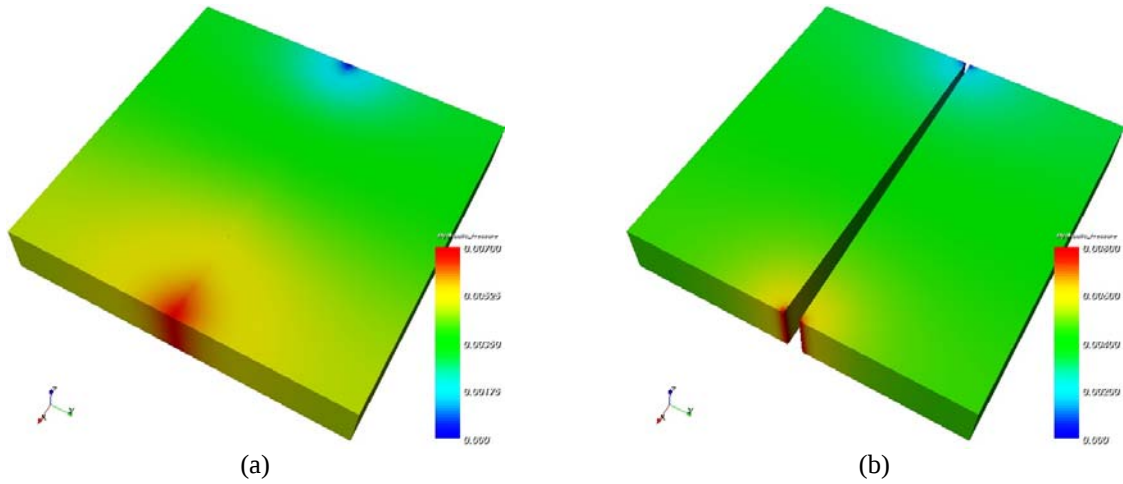


Figure 9.4 – Pressure field distribution over deformed domain for case 3, steps 7 (a) and 8 (b).

In this example, the initial aperture configurations, until step 4, are enough to keep the cycle with a period greater than one step, since the complete closing of the fault is reached at step 5. However, from step 5 ahead, the cycle period is one step, between alternate configurations of close/opened fault, as can be seen in Fig. 10, producing a steady configuration of fault surfaces.

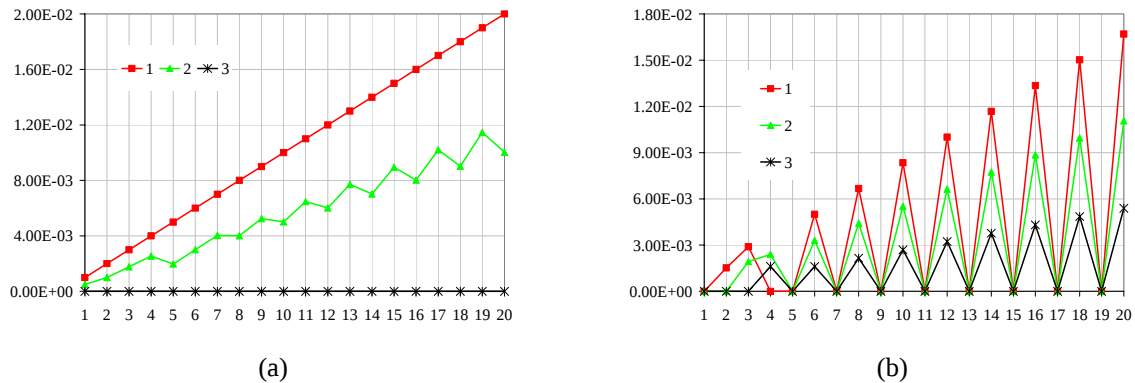


Figure 10 – Pressure versus load step number (a); fault aperture versus step number (b) in points 1, 2 and 3 for case 3.

The complete mechanical analysis was performed in 214 non-linear iterations, generating an average of 193 PCGm iterations per non-linear iteration; hydraulic analysis needed a total of 9,903 PCGh iterations. Figure 11 presents the PCG behavior as iterations counter along load increments and interface elements opened accumulated number for load increment. It can be seen the cyclic configuration of opened/closed fault starting at step 5 and going ahead. The PCGm and PCGh present a behavior similar to the observed in the previous cases, with PCGm driven by the number of interface elements opened and PCGh driven by this number and aperture value as can be observed in Fig. 11(a). The PCGh is influenced by the increasing amplitude as aperture value increases along pressure load steps.

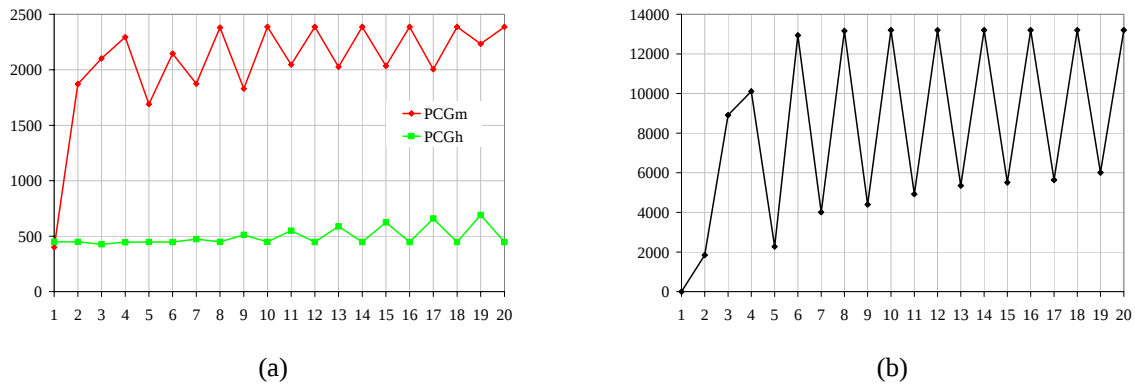


Figure 11 – PCG iterations accumulated number versus step number (a); interface elements opened accumulated number versus step number (b).

## 5. CONCLUSIONS

This work presented a methodology for coupling mechanic and hydraulic problems in fractured porous media. The goal of this methodology is the use of the same topology for both analyses without any need of extra-manipulations of either mesh or results of each problem. This methodology is based on the implementation of three-dimensional prismatic six-noded interface elements with the same topology for mechanical and hydraulic fault modeling.

Results presented a good behavior showing however, a need to refine the study of sharp jumps between fault configurations in order to better capture the entire phenomena. Numerical effects were evaluated and PCG behavior presented along with fault configuration and aperture values revealing interesting fault configurations under different load conditions. The results of the deformed geometries clearly demonstrated the effects of fault aperture over the pressure field. The fault configurations illustrated the formation of hydraulic paths well characterized and the effect of load history over their configurations.

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